THE KNOWLEDGE CREATION IN NETWORK: A COMPARISON BETWEEN FIRM-NETWORK AND NETWORK OF FIRMS

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INTRODUCTION

Two principals' networks are identified in economy. In one hand, firm-network appears to be a centralized network with a fulcrum firm and a set of subcontractors. In this case, information and knowledge are vertically distributed from and toward the central firm. In other hand, network of firms concerns firms which are mostly in same position in terms of competition. It is more decentralized. We find here network as "*milieux locaux*", where information and knowledge (generally tacit) are horizontally distributed.

We consider that the second type of network is more efficient to improve the process of knowledge creation. The framework of our conceptual analysis is based on both the model of knowledge-based creation provided by I. Nonaka (1994) and also on the difference between the J-model and the A-model suggested by Aoki (1986).

The aim of this article is than to demonstrate mathematically the hypothesis that decentralized network is more efficient comparing to a centralized network (firm-network) in terms of knowledge creation. This hypothesis supposes that interactions between firms in a network promote collective learning and organizational knowledge and increase performances. We propose to describe, in terms of mathematical metaphors, the two forms of network in order to compare after their dynamical evolution.

The originality of our approach is based on the assumption that the firm does not adopt optimizing behaviour. In a context of non stochastic uncertainty (contingent uncertainty), the efficiency of the network do not constitute a target. Knowledge held by the firms within the network guide the action and confers than a principle of *satisficing*. To describe these hypotheses mathematically, we base our analysis on the recent developments of the viability theory. (J.-P. Aubin, 1991; 1997).

In the first part, we will present our conceptual framework. The second part is devoted to present the main principles of the viability theory. In the third part, we will examine the model of knowledge creation applied to network topics.

1. CONCEPTUAL FRAMEWORK: FIRM-NETWORK *VERSUS* NETWORK OF FIRMS

The term network refers to a very wide range of inter relations like technical assistance, buyback agreements, patent licensing, franchising, equity joint-venture, etc. (F.J. Contractor and P. Lorange, 1988). As P. Hines (1994) and T. Nishiguchi (1994) pointed out, the most important and a powerful network is the system of the Japanese car industry. Based on the analysis suggested by M. Aoki (1986), the J-firm (Japanese system of network) is more efficient comparing to the A-firm as American firm for transmission of information.

According to P. Cohendet *et al*. (1999), one of the main features of globalised firms (as a firm-network) is to capture part of their fragmented competencies. This evolution represents a new path for international companies. To a large extent, the internationalisation had been characterised up to the mid 1980s by the deployment of activities that had been initially conceived, experienced, tested and developed in the country of headquarters. At that time, the core competencies of companies (including strategic units, research and development resources, etc.) remained in the mother country, and distribution of international activities could be analysed in terms of transfer of an existing knowledge accumulated in the mother country. The main reasons for having sites in different countries were to benefit from existing comparative advantages in terms of given resources or to avoid barriers of entry. In this context, the main obstacles to the co-ordination of activities were due to the existence of possible unequal distribution of information between different activities localised in different places around the world.

Over the last decade, the evolution of global companies took progressively a new direction. On the one hand, the multiplication of mergers, agreements of different types, led to the emergence of islands of knowledge spread all over the world. On the other hand, the development of new information and communication technology that "makes it possible to increase the separability, tradability, divisibility and transportability of information" (C. Antonelli, 1995), facilitated the potential development of new modes of structuring and functioning for firm - network. This context raises new challenges for economic theory: the major problem for a globalised firm to solve is no longer a problem of potential unequal distribution of information, but rather has become a problem of mobilising and integrating fragmented and diversified forms of localised knowledge and competencies. This raises new issues for local competencies in securing competitive advantages. Large global companies are now seeking to build and reinforce their core competencies at the global level by an internal circulation of knowledge between localised sites.

In the network of firms (G. Beccatini, 1979), the territorial environment is important. The concept of industrial district is associated to the productive structure with the social structure and assimilated as industrial *atmosphere.* The district is compared easily with a socio-territory (B. Lecoq, 1991). This approach foresees the analysis of the technological change according to a space. It has the advantage of taking into account the historicity and the irreversibility of technical progress (concept of “technological paradigm” (Nelson & Winter, 1982)).

Without partnership (in particular strategic alliances), SME can thus profit from less formal networks as "les milieux locaux" (R. Planque, 1988). As recalled by P. Veltz (1991), inspired by M.J. Piore & C.F. Sabel (1984), the industrial district is a form of territorialized network of partnership between SME where the behaviours are based on trust and traditions. However, it is important to recall that the fact of sharing cultural values is not enough and that an institutional framework is often necessary to lead to a real co-operation between the companies (J. Szarka, 1990).

We can also consider the Industrial District as a network of firms and even within the network that some leading firms can play a key role in collective leaning processes, (Boari & Lipparini, 1999). The model exposed in this paper can also explain the process of knowledge creation, knowledge sharing and transfer in emerging structure of interfirm relationships. For example Boari and Lipparrini focus on the organisational roles within a network based on the exploitation of the competencies of a selected number of firms and structured processes of knowledge management. "In this case the relational structure for generating and transforming knowledge to partners takes the form of a development project for a specific product". In a certain way the development project generate inside the network of firms, a specific cognitive community where "tacit knowledge is converted to explicit-and there fore codifable-knowledge, triggering original reciprocal processes"(pp 341). These original reciprocal processes illustrate the interactions between cognitive communities within a network of firms supporting an Industrial District.

According to these different features of networks, we suggest, in the same logic than M. Aoki, that the horizontal network is more efficient for knowledge creation because of more interaction and sharing.

2. SOME GLIMPSES ON THE VIABILITY THEORY

The main object of the viability theory is to explain the evolution of a system described by given nondeterministic dynamics and viability constraints, to reveal the concealed regulation laws that allow the system to be regulated, and provide selection mechanisms for implementing them. A common definition of viability is something that can be adapted to constraints in order to be able to live. Viability theory rejects therefore the teleological principle of optimization to focus on viability constraints, on the adaptation of the system. At each moment, a state of the system must respect the viability constraints.

A system is described by state variables and regulation variables. The former correspond to components of the state of the system and can evolve under the impulse of agents. The states evolve according to the regulee, defined as a regulating control or a message detected by the system.

A dynamic law associated to this state and to all regulees defines the velocity of evolution of the state. A second law, called the *a priori retroaction*, describes the constraints to which the regulees are subjected according to the current or past state of the system. It determines all the available regulees, thus clarifying a form of diversity, in the framework of a situation whereby a contingent uncertainty prevails.

Viability theory therefore constitutes a study of the dynamic evolution of a system beyond contingent uncertainty and under viability constraints. The set of states of the system which comply with viability constraints is referred to as *constrained set*. A viable state is an element of this set. A viable evolution is an evolution, which verifies, at each moment, the viability constraints.

This consistency between the dynamics of the system and the viability constraints is important. The initial question, to which the viability theory provides some answers, can be expressed as follows: with each viable state of the macrosystem as an initial state (element of the constrained set), is there at least one viable evolution? (cf. definition 4.1.2 (J.-P. Aubin, 1997, p. 143)). The mathematical resolution of compatibility of constraints and of dynamics of a system is made using the *regulation law*. This law results from the basic viability theorem, which consists in considering that a viable evolution is governed by a viable regulee, obtained through the intermediary of the *regulation map*.

The system is also characterised by three basic principles. On the one hand, its evolution is nondeterministic in the sense that the system is subjected to a contingent uncertainty, and that the existence of a multitude of evolutions according to the different regulees is possible. It also illustrates the idea of flexibility if one considers that it is possible to develop different possible regulees where the environment does not provide for this. On the other hand, the system is subject to viability constraints, which the state of the system must respect "on pain of death". These constraints limit the evolution of the system and determine the domain of constraint in which this evolution must imperatively occur. The third basic principle suggests that the evolution of the system complies with a principle of inertia according to which the regulee evolves only if viability conditions are questioned. If the evolution happens as slowly as possible, this is called the *principle of heavy evolution*.

The notion of the *viability domain* is introduced in order to avoid a situation where all the viable regulees are empty in at least one state of the system. The distinction between a *constrained set* and a *viable domain* is crucial in the sense that it allows the notion of consistency between a dynamics and the constraints to which it is confronted. The theorem of the theory of viability is based on this necessary compatibility: a constrained set is compatible with a macrosystem if and only if it is a viability domain of that macrosystem (cf. theorem 4.1.5. (J.-P. Aubin, 1997, p. 145)).

The mathematical theory of viability uses tools from set-valued analysis such as set-valued maps. It consists in using a set-valued map $\mathbf{C}$, where we associate $\mathbf{x} \in \mathbf{E}$ with a set (possible empty) $\mathbf{C(x)} \subset \mathbf{F}$. On the contrary, in the single-valued analysis, each element $\mathbf{x} \in \mathbf{E}$ is an unique element $\mathbf{f(x)} \in \mathbf{F}$.

This approach enables the treatment of nondeterministic and non-stochastic dynamic models and can also take into account regulation parameters. Its relevance is to widen the spectrum of the study for a solution of the differential equation by replacing it with the notion of differential inclusion.

The advantage of using differential inclusion is thus, on the one hand, to allow the treatment of certain random forms, notably the random form defined by nonstochastic uncertainty. On the other hand, the differential inclusions fit on the case of complex systems, which differ from each other by the absence of control and a variety of possible dynamics.

These are short-sighted systems that do not attempt to attain teleological objectives (*satisfacing behavior*). In contrast to the optimization, only minimal satisfaction requirements are to be respected. The differential inclusion is described by $\mathbf{x}^{'}(\mathbf{t}) \in \mathbf{F}(\mathbf{x}(\mathbf{t}))$, where when at each moment $\mathbf{t}$ the state of the system is $\mathbf{x}(\mathbf{t})$, the set $\mathbf{F}$ determines the set of possible velocities for this state.

3. THE MODEL

First, we describe the network's structure in which firms contribute independently of its repository of knowledge. This constraint will after be introduced in order to develop our problematic. Then, we will introduce the interactions between firms with the help of a connection matrix. Finally, we will describe a network structure which integrates the network's performances into firm behavior.

1. Firm's contributions to the network's performance

Suppose a network composed by **n** firms: $\mathbf{i} = \mathbf{1},...,\mathbf{n}$

In a situation where no constraint of knowledge is imposed on the dynamics of firms contributions to the network, the firm's dynamic behavior denoted by $\mathbf{x}_\mathbf{i}(\mathbf{t}) \in \mathbf{Y}$ at time $\mathbf{t} \geq \mathbf{0}$ (with $\mathbf{Y}$ a finite dimensional vector space such as $\mathbf{Y} = \mathbf{R}^\mathbf{l}$) would be governed by a differential equation of the form, when $\mathbf{x}_\mathbf{i}$ represents the firm's contribution **i**:

$$\text{Let } \mathbf{g}_\mathbf{i} : \mathbf{Y} \to \mathbf{Y} \text{ then } \mathbf{i} = \mathbf{1},...,\mathbf{n} \quad \mathbf{x}_\mathbf{i}^{'}(\mathbf{t}) = \mathbf{g}_\mathbf{i}(\mathbf{x}_\mathbf{i}(\mathbf{t})) \tag{1}$$

The preceding equation describes a sort of "isolated" learning (or self-taught) in the sense that a firm learns without interacting with others. Moreover, his contribution to the network is in this case independent of the knowledge he may have.

In order to integrate the constraint of knowledge into the firm's contribution, we set $\mathbf{M} \subset \mathbf{Y}$ describe firm's knowledge such as:

$$\mathbf{K} \text{ is the set of } \mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_\mathbf{n}) \text{ satisfying } \mathbf{h}(\mathbf{x}(\mathbf{t})) \in \mathbf{M} \tag{2}$$

For example, we consider that in our framework,

$$\forall \mathbf{t} \geq \mathbf{0}, \sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{x}_\mathbf{i}(\mathbf{t}) \in \mathbf{M} \tag{3}$$

$$\text{and } \mathbf{y}(\mathbf{t}) = \sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{x}_\mathbf{i}(\mathbf{t}) = \mathbf{h}(\mathbf{x}(\mathbf{t}))$$

Each firm contributes to the network in a way that the overall contribution satisfies all available knowledge. This equation thus describes the firms' behavior within the constraints of their knowledge by supposing that the constraint, at collective level, is satisfied by independent behavior.

2. Interaction between firms

We will consider now a network structure in which firms interact according to a coordination described with the help of a connection matrix.

If $\mathbf{X = Y^n}$, we propose to connect the associated dynamics to the firm's contributions through a connection matrix $\mathbf{W = (w_i^j) \in L(X, X)}$ (where $\mathbf{L(X, X)}$ is the vector space of matrices). We will consider that the dynamical evolution of the firm's contribution **i** is governed by an evolution law such that:

$$\mathbf{i = 1,...,n} \; ; \; \mathbf{x_i^{'}(t) = \sum_{j=1}^{n} w_i^j(t).g_j(x_j(t))} \quad \text{with} \quad \mathbf{w_i^j \in L(Y, Y)} \qquad (4)$$

The connection matrix determines the interaction between firms who belong to the network. When the connection matrix is the identity matrix noted $\mathbf{1}$, such a network structure, which could be defined as a hollow organization in which firms have no relation, is a structure in which there is no learning from others.

In this configuration, the viability problem consists in determining a regulation, which describes the consistency between firms according to their knowledge.

The rule is that the total of firm contributions $\mathbf{y(t) = h(x(t))}$ remains in the set of knowledge, i.e. it remains inside **M**.

When the total of contributions is almost to leave **M**, problems arise within the network. This situation prevails if the velocity $\mathbf{y^{'}(t)}$ "drives" firm contributions to leave **M**. In this case, the dynamic behavior of the network is no longer viable. One solution consists in supposing $\mathbf{y^{'}(t)}$ is tangent at **M** and remains in the set of knowledge. Mathematically, to clarify the viability theorem, the concept of contingent cone will be introduced (J.-P. Aubin & H. Frankowska, 1990).

Definition 1:

When $\mathbf{K}$ is a subset of $\mathbf{X}$ and $\mathbf{x}$ belongs to $\mathbf{K}$, the contingent cone $\mathbf{T_K(x)}$ to $\mathbf{K}$ at $\mathbf{x}$ is the closed cone of elements $\mathbf{v}$ such as:

$$\mathbf{v \in T_K(x)} \text{ if and only if } \exists \; \mathbf{h_n \to 0+} \text{ and } \exists \; \mathbf{v_n \to v} \text{ such as } \forall \mathbf{n}, \; \mathbf{x + h_n v_n \in K}$$

Thus, starting from **x** in the direction of **v**, the firm's behavior approaches the boundary of **K**, but not too much, insofar, for a countable number of "steps" $\mathbf{h_n > 0}$ and approximate directions $\mathbf{v_n}$, the point $\mathbf{x + h_n v_n}$ lies in $\mathbf{K}$. The contingent cone could be characterized in terms of distances: the contingent cone $\mathbf{T_K(x)}$ to $\mathbf{K}$ at $\mathbf{x}$ is the closed cone of elements $\mathbf{v}$ such that:

$$\mathbf{\liminf_{h \to 0+} \frac{d(x + hv, K)}{h} = 0}$$

We observe that

If $\mathbf{x} \in \mathbf{Int(K)}$, then $\mathbf{T_K(x) = X}$

and that if $\mathbf{K} = \{\bar{\mathbf{x}}\}, \mathbf{T}_{\{\bar{x}\}}(\bar{\mathbf{x}}) = \{\mathbf{0}\}$, we recall that a function $\mathbf{x(.): I \rightarrow K}$ is said to be viable if and only if

$$\forall \mathbf{t} \geq \mathbf{0},\ \mathbf{x(t)} \in \mathbf{K}$$

Then a differential viable function in $\mathbf{K}$ satisfies

$$\forall \mathbf{t} \geq \mathbf{0},\ \mathbf{x}'(\mathbf{t}) \in \mathbf{T_K(x(t))}$$

We can also mention that the contingent cone coincides with the tangent space $\mathbf{T_K(x)}$ of differential geometry when $\mathbf{K}$ is a "smooth manifold". When $\mathbf{K}$ is convex, we prove that the contingent cone coincides with the tangent cone $\mathbf{T_K(x)}$ to $\mathbf{K}$ at $\mathbf{x} \in \mathbf{K}$ of convex analysis, which is the closed spanned by $\mathbf{K - x}$

$$\mathbf{T_K(x)} = \overline{\bigcup_{\mathbf{h} \succ \mathbf{0}} \frac{\mathbf{K - x}}{\mathbf{h}}}$$

The tangent cone is convex. We prove that the contingent cone $\mathbf{T_K(x)}$ is convex whenever $\mathbf{K}$ is sleek at $\mathbf{x}$ that the set-valued map $\mathbf{T_K(.)}$ is lower semicontinuous[1] at $\mathbf{x}$.

We can also apply the viability theorem. If $\mathbf{K = h^{-1}(M)}$ where $\mathbf{h: X \rightarrow Y}$ is a continuously differentiable map such that $\mathbf{h}'\mathbf{(x)}$ is surjective and $\mathbf{M}$ is closed and convex and then:

$$\mathbf{T_K(x) = h'(x)^{-1} T_M(h(x))}$$

The contingent cone has been introduced to associate the firm's behavior and the set of knowledge with the regulation map.

Definition 2:

The regulation map $\mathbf{R_M}$ is the set-valued map:

$$\mathbf{R_M(x)} = \{\mathbf{W \in L(X, X) / h'(x)Wg(x) \in T_M(h(x))}\}$$

where $\mathbf{T_M(y)}$ denotes the contingent cone to $\mathbf{M}$ at $\mathbf{y}$

The regulation map is elaborated by taking into consideration the consistency between firm contributions and their knowledge. In our framework, the regulation map is seen as a sort of behavioral program of the network. It is the formalization of behavioral rules of the network.

[1] Cf. 2.1. *Semicontinuous Set-Valued Maps* and 5.1.2 *Sleek Subsets* in J.-P. Aubin (1991, p. 54 and p. 160).

3. Performances of the network

We couple now together the dynamics of firm contributions and performances of the network defined as a message by firms (regulee).

$$\mathbf{i = 1,...,n \quad x_i'(t) = g_i(x_i(t)) - p(t)} \qquad (5)$$

with $\mathbf{p(t)}$ defined as an message by firms and representing the performances of the network in order that viability constraints of the form

$\forall \mathbf{t \geq 0}$, $\mathbf{h(x(t)) \in M}$ are satisfied.

We consider that firms perceive performances in a same way. The regulee represents here the notion of information redundancy, which is equal for all firms.

The problem of viability consists in introducing a new regulation map taking into consideration the performances defined here as a regulee.

The regulation map $\prod_{\mathbf{M}}$ is the set-valued map such that:

$$\forall \mathbf{x \in K}, \ \prod_{\mathbf{M}}\mathbf{(x) = \{p \,/\, h'(x)(g(x) - p) \in T_M(h(x))\}}$$

The dynamics of the network is viable if and only if firm contributions, according the to scheme of performances, respect this regulation map.

These last two forms of structure and their viability problem present different forms of process of dynamical contributions within the network. We will now demonstrate that the evolution of performances is minimal and identical as the evolution of contributions where individuals do not interact. In order words, we demonstrate that when firms interact, the performance of network is higher.

4. Comparison of dynamical contributions

Insofar as the simplest organization is described by the matrix identity, one can pose in a first stage the problem of the search for a network that minimizes the distance $\|\mathbf{W - 1}\|$. This distance indicates an indicator of connectionist complexity of the network. In a more traditional way, we observe then the modes of network decentralized by the mechanisms of performances. We also pose the problem to make evolve these performances in order to guarantee viability and among these performances, we choose the minimal norm $\|\mathbf{p}\|$. We observe that the regulation by the performances is a particular case of the regulation by the matrix of connections (to some extent summarized in the performances).

One shows thus that the regulation by the simplest network in the sense that the index of connectionist complexity is minimum, among all matrices regulating viable evolution, provides an evolution, which could be controlled by performances. Moreover, the performances have minimal norms.

Recall that in the case of regulation by performances mechanisms, the regulation map is defined by:

$$\forall \mathbf{x} \in \mathbf{K}, \quad \prod_{\mathbf{M}}(\mathbf{x}) = \{\mathbf{p}/\mathbf{h}'(\mathbf{x})(\mathbf{g}(\mathbf{x}) - \mathbf{p}) \in \mathbf{T}_{\mathbf{M}}(\mathbf{h}(\mathbf{x}))\}$$

The viability theorem involves that the performances $\mathbf{p(t)}$ regulating viable solutions are given by the regulation law:

$$\mathbf{p(t)} \in \prod_{\mathbf{M}}(\mathbf{x(t)})$$

For determining feedback performances, we can select some innovative performances of the regulation map like $\varpi^0(\mathbf{x}) \in \prod_{\mathbf{M}}(\mathbf{x})$ with minimal norm. Viable solutions obtained with feedback performances are the slow viable solutions.

When $\mathbf{B} \in \mathbf{L(X, Y)}$ is surjective, its orthogonal right inverse[2] is equal to

$$\mathbf{B}^+ = \mathbf{B}^*(\mathbf{BB}^*)^{-1}$$

that we can supply $\mathbf{Y}$ with the final norm $\mu^{\mathbf{B}}$ defined by $\mu^{\mathbf{B}}(\mathbf{z}) = \|\mathbf{B}^+\mathbf{z}\|$ and that we denote by $\pi_{\mathbf{K}}^{\mathbf{B}}$ the projector of best approximation onto the closed subset $\mathbf{K}$ for this final norm.

The unique solution $\bar{\mathbf{x}}$ to the minimization problem[3].

$$\inf_{\mathbf{Bx} \in \mathbf{K}+\mathbf{v}} \|\mathbf{x} - \mathbf{u}\|$$

is equal to

$$\bar{\mathbf{x}} = \mathbf{u} - \mathbf{B}^+(\mathbf{1} - \pi_{\mathbf{M}}^{\mathbf{B}})(\mathbf{Bu} - \mathbf{v})$$

Moreover, when $\mathbf{M}$ is sleek (and in particular convex), then its tangent cones $\mathbf{T}_{\mathbf{M}}(\mathbf{y})$ are convex (J.-P. Aubin & H. Frankowska, 1990). Since the polar cone to the contingent cone $\mathbf{T}_{\mathbf{M}}(\mathbf{y})$ is the normal cone $\mathbf{N}_{\mathbf{M}}(\mathbf{y})$, we can determine the solution to the minimization problem

$$\inf_{\mathbf{Bx} \in \mathbf{T}_{\mathbf{M}}(\mathbf{y})+\mathbf{v}} \|\mathbf{x} - \mathbf{u}\|$$

is equal to

$$\bar{\mathbf{x}} = \mathbf{u} - \mathbf{B}^+(\mathbf{1} - \pi_{\mathbf{T}_{\mathbf{M}}(\mathbf{y})}^{\mathbf{B}})(\mathbf{Bu} - \mathbf{v}) = \mathbf{u} - \mathbf{B}^*\pi_{\mathbf{N}_{\mathbf{M}}(\mathbf{y})}^{\mathbf{B}^*}(\mathbf{BB}^*)^{-1}(\mathbf{Bu} - \mathbf{v})$$

where $\pi_{\mathbf{N}_{\mathbf{M}}(\mathbf{y})}^{\mathbf{B}^*}$ denotes the projector onto the normal cone $\mathbf{N}_{\mathbf{M}}(\mathbf{y})$ when the dual $\mathbf{Y}^*$ is supplied with the dual final norm $\|\mathbf{B}^*\mathbf{q}\|$.

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[2] Cf. Definition 10.1.2 in J.-P. Aubin (1997, p. 348).
[3] Cf. 10.1.1. *Projections onto Inverse Images of Convex Sets* in J.-P. Aubin (1997, p. 348)

Slow solutions to this dynamical system to our problem are given by

Proposition 1:

Let assume that the map $\mathbf{g}$ is continuous and bounded and that the knowledge map $\mathbf{h}$ is continuously differentiable and satisfies the uniform surjectivity condition:

$$\forall \mathbf{x} \in \mathbf{K},\ \mathbf{h}'(\mathbf{x}) \text{ is surjective } \mathbf{\&} \ \sup_{\mathbf{x}\in\mathbf{K}} \left\| \mathbf{h}'(\mathbf{x})^{+} \right\| \prec +\infty \qquad (6)$$

and that $\mathbf{M}$ is closed convex. Then the slow viable solution of the dynamic

$$\mathbf{x}'(\mathbf{t}) = \mathbf{g}(\mathbf{x}(\mathbf{t})) - \mathbf{p}(\mathbf{t})$$

subjected to the viability constraints

$$\forall \mathbf{t} \geq \mathbf{0},\ \mathbf{h}(\mathbf{x}(\mathbf{t})) \in \mathbf{M}$$

is the solution to the differential equation

$$\mathbf{x}'(\mathbf{t}) = \mathbf{g}(\mathbf{x}(\mathbf{t})) - \varpi^{\mathbf{0}}(\mathbf{x}(\mathbf{t}))$$

where

$$\varpi^{\mathbf{0}}(\mathbf{x}) = \mathbf{h}'(\mathbf{x})^{+}(\mathbf{1} - \pi^{\mathbf{h}'(\mathbf{x})}_{\mathbf{T_M}(\mathbf{h}(\mathbf{x}))})\mathbf{h}'(\mathbf{x})\mathbf{g}(\mathbf{x}) = \mathbf{h}'(\mathbf{x})^{*}\pi^{\mathbf{h}'(\mathbf{x})^{*}}_{\mathbf{N_M}(\mathbf{h}(\mathbf{x}))}(\mathbf{h}'(\mathbf{x})\mathbf{h}'(\mathbf{x})^{*})^{-1}\mathbf{h}'(\mathbf{x})\mathbf{g}(\mathbf{x}) \qquad (7)$$

Proof:

Under the criterion of lower semicontinuous condition, the regulation map $\prod_{\mathbf{M}}$ is lower semicontinuous because the set-valued map $\mathbf{T_M}(.)$ is semicontinuous and the surjectivity assumption (6) implies assumption (11.2.1) of proposition (11.2.10) in J.-P. Aubin (1997, p. 363 and 368). Slow viable solution exists and is obtained by the performances $\varpi^{\mathbf{0}}(\mathbf{x})$ of minimal norm among the performances satisfying $\mathbf{h}'(\mathbf{x})\mathbf{p} \in \mathbf{T_M}(\mathbf{h}(\mathbf{x})) - \mathbf{h}'(\mathbf{x})\mathbf{g}(\mathbf{x})$. Proposition (10.1.4.) in J.-P. Aubin (1997, p. 349) implies formula (7).

We will now compare the dynamics of contributions dictated by the connection matrices which describe coordination between firms in a network with those obtained by independent contributions to performances.

We compare the dynamics $\mathbf{x}'(\mathbf{t}) = \mathbf{g}(\mathbf{x}(\mathbf{t})) - \mathbf{p}(\mathbf{t})$ which respects minimal cognitive constraints $\forall \mathbf{t} \geq \mathbf{0}$, $\sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \mathbf{x_i}(\mathbf{t}) \in \mathbf{M}$, to that whereby firms are connected by a connection matrix $\mathbf{W} = (\mathbf{w}_{\mathbf{i}}^{\mathbf{j}}) \in \mathbf{L}(\mathbf{X},\mathbf{X})$ which can be written as follows:

$$\mathbf{x}'(\mathbf{t}) = \mathbf{W}(\mathbf{t})\mathbf{g}(\mathbf{x}(\mathbf{t})) \qquad (8)$$

We propose to demonstrate that the dynamics linked to $\mathbf{x}^{'}(\mathbf{t}) = \mathbf{g}(\mathbf{x}(\mathbf{t})) - \mathbf{p}(\mathbf{t})$ and that linked to $\mathbf{x}^{'}(\mathbf{t}) = \mathbf{W}(\mathbf{t})\mathbf{g}(\mathbf{x}(\mathbf{t}))$ are identical.

Whenever $\mathbf{p} \in \mathbf{X}^*$ (dual of the matrix **X**) and if $\mathbf{y} \in \mathbf{Y}$, we denote by $\mathbf{p} \otimes \mathbf{y} \in \mathbf{L}(\mathbf{X}, \mathbf{Y})$ (tensor product[4]) the rank one linear operator defined by $\mathbf{x} \to (\mathbf{p} \otimes \mathbf{y})(\mathbf{x}) = \langle \mathbf{p}, \mathbf{x} \rangle \mathbf{y}$, the matrix of which is

$$(\mathbf{p}^{\mathbf{i}} \mathbf{y}_{\mathbf{j}})_{\substack{\mathbf{i}=1,\ldots,\mathbf{m} \\ \mathbf{j}=1,\ldots,\mathbf{n}}}$$

Both finite dimensional vector spaces **X** and **Y** are identified with their duals. We observe then:

Proposition 2:

Let $\Gamma : \mathbf{X} \to \mathbf{X}$ be any continuous map such that

$$\inf_{\mathbf{x} \in \mathbf{K}} \langle \Gamma(\mathbf{x}), \mathbf{g}(\mathbf{x}) \rangle > \mathbf{0}$$

Then a connection matrix of the form

$$\mathbf{W}(\mathbf{x}) = \mathbf{1} - \frac{\Gamma(\mathbf{x})}{\langle \Gamma(\mathbf{x}), \mathbf{g}(\mathbf{x}) \rangle} \otimes \varpi(\mathbf{x})$$

where the values $\varpi_{\mathbf{ij}}(\mathbf{x})$ are equal to

$$\omega_{\mathbf{ij}}(\mathbf{x}) = \delta_{\mathbf{i},\mathbf{j}} - \frac{\Gamma(\mathbf{x})_{\mathbf{i}} \varpi(\mathbf{x})_{\mathbf{j}}}{\langle \Gamma(\mathbf{x}), \mathbf{g}(\mathbf{x}) \rangle}$$

belongs to $\mathbf{R}_{\mathbf{M}}(\mathbf{x})$ if and only if $\varpi(\mathbf{x})$ belongs to $\prod_{\mathbf{M}}(\mathbf{x})$ and then the viable solutions to $\mathbf{x}^{'} = \mathbf{W}(\mathbf{x})\mathbf{g}(\mathbf{x})$ and $\mathbf{x}^{'} = \mathbf{g}(\mathbf{x}) - \varpi(\mathbf{x})$ are identical.

Proof:

We observe that

$$\mathbf{W}(\mathbf{x})\mathbf{g}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) - \frac{\langle \Gamma(\mathbf{x}), \mathbf{g}(\mathbf{x}) \rangle}{\langle \Gamma(\mathbf{x}), \mathbf{g}(\mathbf{x}) \rangle} \varpi(\mathbf{x}) = \mathbf{g}(\mathbf{x}) - \varpi(\mathbf{x})$$

Then the differential equation $\mathbf{x}^{'}(\mathbf{t}) = \mathbf{W}(\mathbf{x})\mathbf{g}(\mathbf{x})$ and $\mathbf{x}^{'} = \mathbf{g}(\mathbf{x}) - \varpi(\mathbf{x})$ are the same.

If $\mathbf{W}(\mathbf{x}) \in \mathbf{R}_{\mathbf{M}}(\mathbf{x})$ then $\mathbf{h}^{'}(\mathbf{x})\mathbf{W}(\mathbf{x})\mathbf{g}(\mathbf{x}) = \mathbf{h}^{'}(\mathbf{x})\mathbf{g}(\mathbf{x}) - \mathbf{h}^{'}(\mathbf{x})\varpi(\mathbf{x}) \in \mathbf{T}_{\mathbf{M}}(\mathbf{h}(\mathbf{x}))$ and the viability conditions are also the same.

■

[4] Cf. *Tensor Products of Linear Operators* in J.-P. Aubin (1997, p. 353).

The regulation by connection matrices of the form

$$\mathbf{W(x) = 1 - \frac{\Gamma(x)}{\langle \Gamma(x), g(x) \rangle} \otimes \varpi(x)}$$

is the same as a performance mechanism whenever $\mathbf{g(x) \neq 0}$ for each $\mathbf{x \in K}$.

In this way, the problem of no interaction between firms is then to find connection matrices as close as possible to the identity matrix.

Proposition 3:

Suppose $\mathbf{g}$ is a continuous and bounded map and the map of firm contributions $\mathbf{h}$ is continuously differentiable and satisfies the uniform surjectivity condition:

$$\forall \mathbf{x \in K}, \ \mathbf{h^{'}(x)} \text{ is surjective and } \mathbf{\sup_{x \in K} \frac{\| h^{'}(x)^{+} \|}{\| g(x) \|} < +\infty} \qquad (9)$$

Suppose that $\mathbf{M \subset Y}$ is a closed convex subset. Then the set of knowledge $\mathbf{K = h^{-1}(M)}$ is viable for equation (8), denoting the network when firms are interconnected if and only if for each $\mathbf{x \in K}$, the image $\mathbf{R_M(x)}$ of the regulation map is not empty.

The evolution resulting from no interactions subject to the viability constraint:

$$\forall \mathbf{t \geq 0}, \ \mathbf{h(x(t)) \in M}$$

is governed by the differential equation:

$$\mathbf{x'(t) = W^0(x(t))g(x(t))}$$

where

$$\begin{aligned} \mathbf{W^0(x)} &= \mathbf{1 - \frac{g(x)}{\|g(x)\|^2} \otimes h^{'}(x)^{+}(1 - \prod\nolimits_{T_{M(h(x))}}^{h^{'}(x)}) h^{'}(x)g(x)} \\ &= \mathbf{1 - \frac{g(x)}{\|g(x)\|^2} \otimes h^{'}(x)^{*} \prod\nolimits_{N_{M(h(x))}}^{h^{'}(x)^{*}} \left(h^{'}(x)h^{'}(x)^{*}\right)^{-1} h^{'}(x)g(x)} \end{aligned} \qquad (10)$$

The slow viable solutions to the dynamics (7) and (8) are the same. In others words, the dynamics of a network (individuals do not interact) is equivalent to the dynamics of an organizational structure where the performances of the network are included into the firm contributions and are minimal.

Proof:

The regulation map could be denoted as following:

$$\mathbf{R_M(x)} = \left\{ \mathbf{W} \in \mathbf{L(X,X)} / (\mathbf{g(x)} \otimes \mathbf{h}'(\mathbf{x}))\mathbf{W} \in \mathbf{T_M(h(x))} \right\}$$

Then the viable solutions are regulated by the regulation law:

$$\left(\left(\mathbf{g(x(t))}\right) \otimes \left(\mathbf{h}'(\mathbf{x(t)})\right)\right)\mathbf{W(t)} \in \mathbf{T_M}\left(\mathbf{h(x)}\right)$$

Insofar as the map $\mathbf{g(x)} \otimes \mathbf{h}'(\mathbf{Wx})$ is surjective from the space $\mathbf{L(X,X)}$ of connection matrices to the firm knowledge space $\mathbf{Y}$, then

$$\left\| \left(\mathbf{g(x)} \otimes \mathbf{h}'(\mathbf{x})^{+}\right) \right\| = \frac{\left\| \mathbf{h}'(\mathbf{x})^{+} \right\|}{\left\| \mathbf{g(x)} \right\|}$$

Then, assuming that $\mathbf{R_M}$ is lower semicontinuous, the viable connection matrices near the identity matrix are the solution of $\mathbf{W^0}$ minimising the distance $\|\mathbf{W} - \mathbf{1}\|$ among the connection matrices satisfying the viability constraint.

According to the theorem of orthogonal projections on subset of matrices (cf. theorem 10.3.6. in J.-P. Aubin (1997) p. 358), the map $\mathbf{g(x)} \otimes \mathbf{h}'(\mathbf{Wx})$ is surjective from the space $\mathbf{L(X,X)}$ of connection matrices to the firm knowledge space $\mathbf{Y}$. The solution is given by (10).

CONCLUSION

We showed that the dynamics of behaviour where firms interact is more efficient as the firm-network. Certain points could be developed; in particular the determination of selection mechanisms for the behaviour of the firm constitutes an important question. It would be also interesting to treat the interactions between the firms by introducing a matrix of connection at the level of knowledge. A possibility is to base this analysis to the concept of communities which are mainly developed within the firm and could have some application in network schemes. All these questions could be analysed with viability theory and allow to further consolidate the relevance of this mathematical theory to analyse economic issues.

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